12-tone score and
just intonation

♯ & ♭ are not necessary at all to music

volume 1

New musical grammar / New notation

Sumio Natsuyama

Introduction 1

＃&♭ are not necessary at all to music

New musical grammar/New notation (Volume 1)

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（http://km87290.web.fc2.com/htdocs/English/English.html）

Introduction

It is not so preferable to use ＃&♭ in music. Though ＃&♭ are utilized traditionally as a matter of course in music, they had better not be used originally. As ＃&♭ make music complicated, they are not suitable to music. It may be very disappointing, but these marks make music rather difficult to understand. As they are unsuitable to music, they should not be used completely at all in music.

Some people may think that ＃&♭ must be used in case of just intonation. But I will show that even in case of just intonation they need not be used in this report. We can do it easily without using ＃&♭, even in case of equal temperament or just intonation. As there are these marks in music, music has some special peculiarity, and musical scores become more complicated. There is another way in music, which is natural and may become a wider acceptance in many people.

Some explanations may be necessary, as I say that music has some special peculiarity owing to ＃&♭. We can see this peculiarity clearly in music key. There is a special unbalance of key in the number of music. For example, there are a lot of music scores with G key, as compared with music scores with G♭ key which is only a half-tone lower. Why are there so many numbers of scores with G key, and are there so few numbers of scores with G♭key？ Does this depend on music tonality? The answer is "No". The numbers of scores with G key and with G♭key do not depend on music tonality. They depend only on the difficulties of musical grammar, which is peculiar. We use only one ＃ in G key, and 6 ♭s in G♭key.

Music beginners often play music of G key and never play music of G♭ key. Music teacher does not make pupils play G♭ key music at first. But G♭ key music playing is easier than that of G key music, as many black piano keyboards are used in G♭ key music. In the piano playing we can easily play the music in which many black keyboards are used, as the piano has fewer black keyboards compared with white keyboards. Scores that have many black keyboards are simple and easy to learn. Therefore it is better to learn music at the beginning, whose key has more black keyboards.

Some piano teachers may make pupils play "five funny fingers （Floh walzer）" at first in which many black keyboards are used. But in this case they can not teach scores at the same time. In learning music, both playing the instruments and reading scores are necessary. Only playing the instruments is not a formal learning.

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＃&♭ are good neither for equal temperament nor for just intonation. ＃&♭ are half measures, and do not fit for both. In case of equal temperament it is easily understood for one, who is not so well acquainted with music, that ＃&♭ are unnecessary to music, as there are only 12 tones in one octave. So only 12 notes are sufficient for an octave of 12 tones. Owing to ＃&♭, "enharmonic notes" are generated and it is clear that these are not necessary. On the other hand, in case of just intonation ＃&♭ do not make it convenient to know how just intonation is formed. By using ＃&♭ we cannot understand its formation easily. ＃&♭ make just intonation more complicated and we cannot get it simply. So ＃&♭ disturb eventually many people from approaching just intonation. There are not so many people who understand 53 tones of just intonation. Those who know the 53 tones are not so acquainted with them, as they seem more complicated. But 53 tones of just intonation are simple and they become easy to understand, if we do not use ＃&♭ in music. This shows that ＃&♭ are not suitable to music. We had better have a new musical grammar or musical notation in which ＃&♭ are not used.

In this home page new and simple notation method is presented, in which ＃&♭ are not used at all. By modifying a little the conventional method, we can get one that is simple and easy to understand.

Though few people are interested in contemporary music, this new method is also suited for contemporary music in which twelve-tone music and quarter tone or sixth tone are applied.

And finally newly proposed useful equal temperament is presented which can be used both in equal temperament and just intonation, and becomes universal. By this equal temperament, musical transposing to any key is very simple and free. This is 144 equal temperament and this can be easily applied to contemporary music, as it contains not only half tone but also quarter tone, sixth tone and eighth tone.

When you read this book, please keep it in mind that all of the figures or tables have one special important rule in its content, though there are a lot of figures or tables and at a glance they seem complicated. When you see these contents with that rule, they become clear and easy.

The content of this book is introduced in the next homepage. Refer to this homepage, as it has also English version.

（http://km87290.web.fc2.com/htdocs/English/English.html）　（Homepage of Sumio Natsuyama)

Content 1

Content (Volume 1)

Content 2

(Content of volume 2 and volume 3 is in the last pages)

If you want to get interval value, go to the next calculation page
(http://km87290.web.fc2.com/htdocs/English/page20.html)

Chapter 1 12-tone score

☆☆ Items changed newly ☆☆

In order to make music easy, I propose to change the musical grammar newly. Speaking in a word, it is that conventional 7 pitch tones are to be increased to 12 pitch tones. This means that new 12 names are newly decided as shown in the next table. As these 12 tones become independent from now on, this change has a very big impact on conventional grammar.

＝ Conventional 7 pitch names ＝

	1	2	3	4	5	6	7
pitch name	C	D	E	F	G	A	H

＝ New 12 pitch names ＝

	1	2	3	4	5	6	7	8	9	10	11	12
new pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB

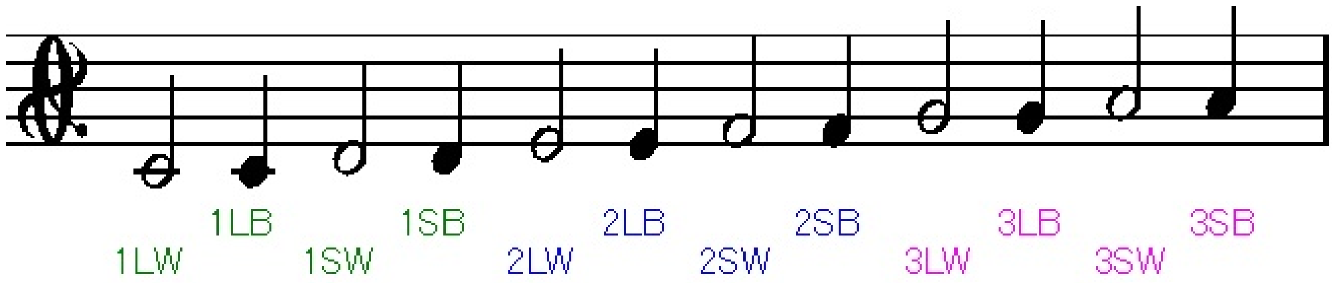
In the conventional method in which ♯& ♭ are used, C major is treated as special key and C major key has only special meaning in all the keys. Therefore all other keys are mere appendices of C major, belonging to that key. In mediaeval era before Bach, when only C major had special meaning, there may be no problem. But now C major has not special position any more. Even in classic music there are few C major keys. It is not good that C major is treated as very important key and other keys are the sacrifices of C major.

These amendments cause various changes automatically in related notation. From now on these changing points are described one by one in orderly sequence.

As new pitch names mentioned above may not yet be familiar, I explain them in more detail. Conventionally a white note means a whole note and a half note and a black note means the note whose length of time is shorter than a white note. This point is different in new notation. The length of time of a white note and a black note is quite the same. But the pitch of the tone is different between the two notes. The pitch of a black note is a half tone higher than that of a white note. And a staff is different a little from the conventional staff.

The new staff is as follows. The pitch of each note shown in the next figure is raised every half tone.

＝ Figure of 12 tones ＝



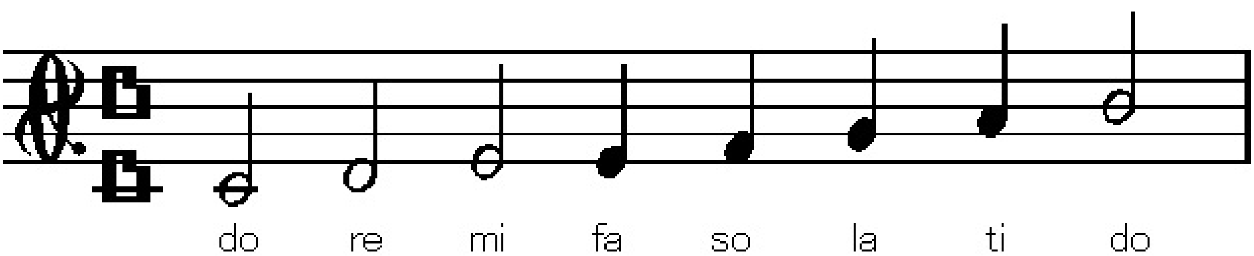
As shown above 12 tones are expressed as notes that exist among 3 lines and 3 spaces. An octave is expressed within 3 lines and 3 spaces. 3 lines or 3 spaces raise one octave. Similarly 3 lines or 3 spaces lower one octave. The new mark on the left means clef. L means line and S means space. Similarly W means white and B means black.

For example the notation of C major is in the next figure using new 12 tones.

C major	do	re	mi	fa	so	la	ti	do
new pitch name	1LW	1SW	2LW	2LB	2SB	3LB	3SB	1LW

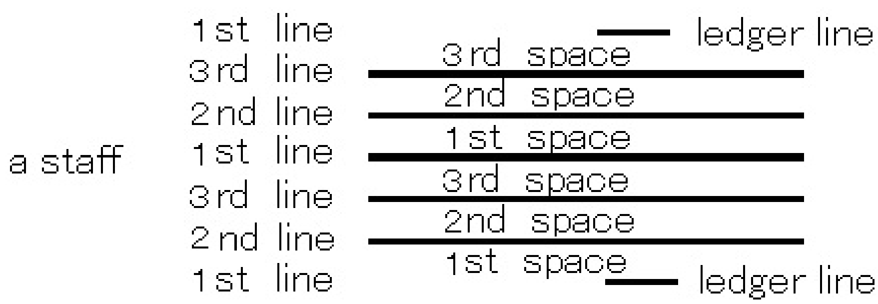
The score is similarly in the next figure.

＝ C major scale ＝



The new symbol located next of clef on the left is key signature which indicates C major. How to count new lines and new spaces in a staff is shown below.

＝ How to count lines and spaces in a staff ＝



This numbering is due to the cycle, being made by 3 lines and 3 spaces, which is described above.

☆☆ 12-tone score which is the basis ☆☆

This type of score is named 12-tone score. All notes are decided only by 12 tones in an octave. In all the description of this report, 12-tone score becomes always a basis.

☆☆ Already published 12-tone score book ☆☆

It may be difficult to understand, if the notation method of score is explained in sentences. If you see real score, you may understand it well. The books of 12-tone score are already published. 12-tone score is desired to be read beforehand for early understanding of this book.

①『この楽譜なら、音楽はやさしい！』, which means in Japanese that music becomes easy, if this score is used.（the author: Sumio Natsuyama; publisher: Souei Shuppan）(ISBN4-434-04101-0 C3073)

②『音楽が身近になるやさしい新楽譜』, which means in Japanese that the new score by which music becomes familiar and easy.（the author: Sumio Natsuyama; publisher: Hon no Izumisha）(ISBN4-88023-937-2 C0073)

Though these two books are written in Japanese, you can use scores in the books, as scores are global language.

If you want to get them, you can buy them at Amazon by net. For example you can see them by the next "URL"
http://www.amazon.co.jp/s/ref=nb_sb_noss?__mk_ja_JP=%83J%83%5E%83J%83i&url=search-alias%3Daps&field-keywords=%89%C4%8ER%90%9F%95v&x=8&y=19

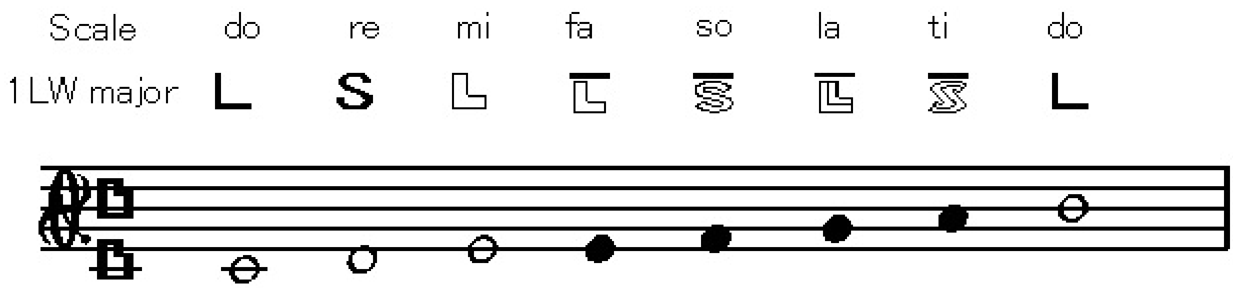
These books are written for keyboard instruments. In these books are listed La Viollette (Louis Streabbog), Dolly's Dreaming and Awakening (Theodor Oesten), Farewell etude op.10 No.3 (F.F.Chopin), Waltz op. 69 No. 2 (F.F.Chopin), Voices of Spring (Johann Strauss II) and others, together with folk songs and marches. All 12 keys of music are listed in these scores. This means that it is devised to be able to play scores in which all kinds of key signatures are used in these books.

Based on these books, Pythagorean intonation and just intonation are described in this book not for keyboard instruments but for just intonation instruments, whose sounds in an octave are more than 12 tones and that are mainly stringed instruments.

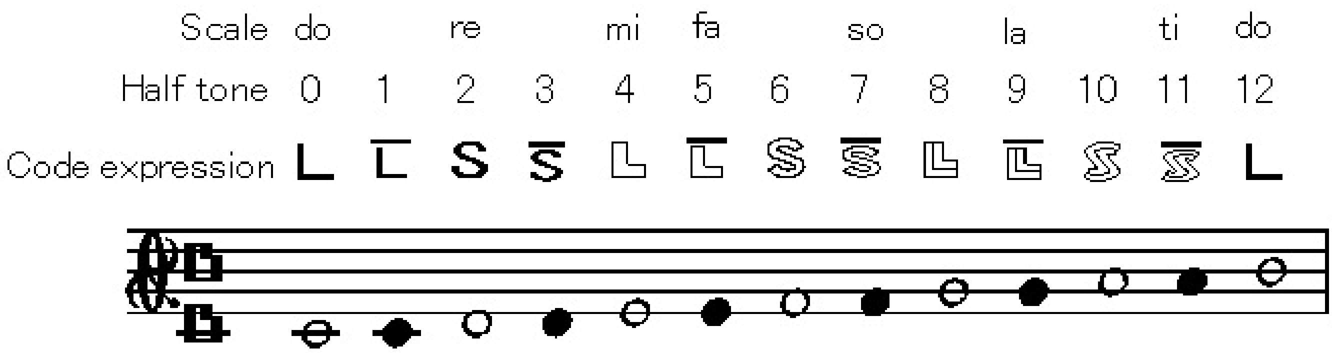
☆☆ What is 12-tone score ? ☆☆

Though 12-tone score can be understood with reading 2 books already published as described above, it is also explained simply here. The scale of 12-tone score is shown in the figure of next page.

＝ Scale of 1LW (C) major 12-tone score ＝



In case of 12 tones the score becomes as shown in the next figure.



☆☆ Example of the score ☆☆

The next is the example of 12-tone score. As there are no accidentals, it is simple. The scores are “twinkle twinkle little star” and Chopin’s prelude.

A part of the score of Chopin’s prelude is presented in the next figure.

As the pitch tone can be read directly from the note, this becomes very easy score, when you become familiar with seeing this type of notes.

The score is conventionally one type. But this new change makes it possible to write different type scores, depending on equal temperament or just intonation. And it makes also possible to clear the differences that are vague till now. These are described precisely in detail later.

☆☆ How to write notes of the instruments which have more than 12 tones in an octave ☆☆

12-tone score is mainly used for keyboard instruments. And it is necessary to write scores for the instruments that have more than 12 tones.

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Before explaining it, it is necessary to consider how it is up to now. It is important that we know the score for just intonation is very special. And there is no score written only for music of just intonation till now, as it is necessary for this score to make the tone strictly clear in the just intonation tones. Actually the notation for just notation is already proposed. But it is not put to practical use.

And there are other problems that how it is played precisely with just intonation. According to music theory there are 53 tones of just intonation. And it is rare that in every key just intonation tones are only utilized and other tones are never used. (If Pythagorean intonation tone is used, it is different from 53 just intonation tone.) This problem depends upon the player himself. Therefore there are many cases in which the scores need not be written with just intonation, even if the score should be played with just intonation. The details of just intonation are described later. Here I emphasize only that there are some inherent problems in the notation of just intonation.

☆☆ Pitch name and code expression ☆☆

The pitch name expressed by code can be understood intuitively. Code expression method is described below.

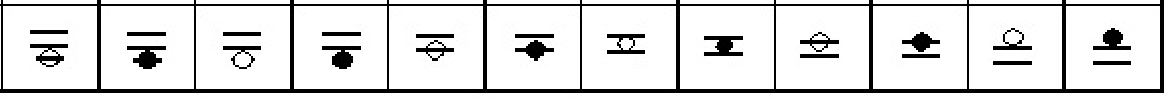
pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB
	L	L̄	S	S̄	𝕃	𝕃̄	𝕊	𝕊̄	𝕃	𝕃̄	𝕊	𝕊̄

There is a cycle of 3 lines and 3 spaces in a new staff. Each line and each space have line tone and space tone, respectively. A line tone is a tone on the line and a space tone is a tone on the space. Line tone is L and space tone is S. A white tone is a tone of the white note and a black tone is a tone of the black note. On the top of mark of a black tone a bar is attached for discrimination from a white tone.

The first line and the first space are a single character L and S. The second line and the second space are a double character 𝕃 and 𝕊. The third line and the third space are a triple character 𝕃 and 𝕊.

Not only such code expression but also the note expression is shown in the next table, for further explanation. This means a note on a kind of a staff whose upper 3 lines are omitted. For example the note figure of 1LW tone (L) in the left side is expressed by the new concept of the staff explained above. Namely the first line, the second line and the third line are plotted in the figure and this white circle of note is 1LW. Similarly in the figure of 1SW tone (S) only the second line and the third line are plotted and the white circle of note is at the place of the first space. The notes of other tones are similarly plotted and 2 lines mean only the second line and the third line.

By showing the second line and the third line, the position of the note, which shows the pitch of the tone, is indicated.

pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB
	L	L̄	S	S̄	𝕃	𝕃̄	𝕊	𝕊̄	𝕃	𝕃̄	𝕊	𝕊̄
note												

☆☆ Syllable name modification ☆☆

As previously described pitch names are increased from 7 tones to 12 tones. Though pitch names are increased by this modification, syllable names have only 7 tones of do re mi fa so la ti do. This is because the scale is 7 tones of do re mi fa so la ti do. If we increase syllable names also to 12 tones, it becomes very convenient.

pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB	1LW
	L	L̄	S	S̄	𝕃	𝕃̄	𝕊	𝕊̄	𝕃	𝕃̄	𝕊	𝕊̄	L
syllable name	do	new name1	re	new name2	mi	fa	new name3	so	new name4	la	new name5	ti	do

In the table above the following new names are given as syllable names.

pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB	1LW
	L	L̄	S	S̄	𝕃	𝕃̄	𝕊	𝕊̄	𝕃	𝕃̄	𝕊	𝕊̄	L
syllable name	do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do

For example dor tone means that the tone locates between do tone and re tone.

☆☆ 12-tone doremi song ☆☆

The song is made in which all tones of 12 syllable names are used. This is a song for reminding 12 syllable names. This is named 12-tone doremi song. Please refer to this song.

Though, as there is no syllable name dor, it is conventionally expressed as “C♯D♭”, we can stop it from now on. There is no inevitability to express it in that way. Though some people may think that it is necessary to express it in case of just intonation, it is not necessary to do so.

(note: As in case of equal temperament dor tone is limited to one tone, it is not necessary to be shown above. In case of just intonation there are several tones between do tone and re tone, which are a syntonic comma different among each other. But there is one tone in the scale in which just intonation is applied there.

If all tones which are a syntonic comma different need to be expressed, it is not suitable to express as C♯D♭, which means that there are only two tones, as there are actually more tones than 2.

Up to now it was only expressed by habit and this expression has no practical meaning.)

☆☆ Musical grammar of 12-tone score ☆☆

Though it is now necessary to explain the details of musical grammar of 12-tone score, it may be incomprehensible, unless you do not read the already published books, which are introduced formerly. Therefore it is not described here and its details are explained separately in page 83 of volume 2 as musical grammar of 12-tone score. Please refer to it.

☆☆ When accustomed to play with using 12-tone score ☆☆

12-tone score is a handy score with which keyboards and other instruments can be easily played. With using this score we can play any key music in the same way. There are not differences of difficulties of playing music among all keys. It seems as if all the keys are C major and we have only to play C major. Moreover it is easier than C major, as there is no accidental mark at all. Therefore we do not mind the key in playing, when we are accustomed to play the score.

The man, who has experience of playing for example such pieces of music as Chopin's "Raindrop" prelude op. 28, no. 15, Mozart's 12 variations "Ah, vous dirai-je, maman" K. 265, Mendelssohn's "Spring Song" op.62 no.6, Baranowska's "A Maiden's Prayer" or Ponchielli's "Dance of the Hours", can immediately say what keys they are, even though he plays well or not. The key must be the most important concerning subject for playing, apart from the problem of tonality.

The 12-tone score, however, is different, as the key is not the important subject for playing. (We can know always naturally the key, as there is key signature in the 12-tone score and chords used, without noticing it.) And we also have experience of easiness in key transposing. Therefore we do not remember after the playing whether the score had key transposing or not, or how many times key transposing occurred, when we do not mind it.

As mentioned above 12-tone score makes player completely free from the burden of key. So player needs not mind the key practically at all, if it is limited only to the playing.

☆☆ How to count interval ☆☆

If we do not use ♯& ♭, how to count interval is changed completely. It becomes unnecessary to count it with using "degree". Details are described later, and here I say that interval is counted only by using the numbers of half tones contained between the 2 different tones. For example interval of a half tone is 100 cents and interval of two half tones is 200 cents. Therefore speaking conversely, 100 cents are a half tone interval and 200 cents are two half tone interval. And 1200 cents of an octane is naturally 12 half tones. This is very simple compared with the conventional method.

Chapter 2 Temperament which has more than 12 tones

☆☆ Tone array ☆☆

Until now 12-tone score is explained, and this means that there are only 12 tones in an octave. Though they are limited to 12 tones in keyboard instruments, other instruments have more than 12 tones in an octave. From this section more tones than 12 are treated, and this means that temperament or intonations are taken into account.

☆☆ Conventional tone array ☆☆

Till now the following tone array is known as that of just intonation. There are various figures in the expression of the array decided by music theory. Though some of them are upside down, all their contents are just the same.

				10 eses	41 heses	19 fes	50 ces	28 ges	6 des	37 as	15 es	46 b
			49 ces	27 ges	5 des	36 as	14 es	45 b	23 f	1 c	32 g	
		35 as	13 es	44 b	22 f	0 c	31 g	9 d	40 a	18 e		
	21 f	52 c	30 g	8 d	39 a	17 e	48 h	26 fis	4 cis			
7 d	38 a	16 e	47 h	25 fis	3 cis	34 gis	12 dis	43 ais				
24 fis	2 cis	33 gis	11 dis	42 ais	20 eis	51 his	29 fisis					

I present a new figure that is different from the conventional style and this array is shown in the figure below. Though the content is the same, a part of the arrangement is modified. This array is point symmetry whose center is zero point that is origin. This improvement makes regularity clear and makes it easier to understand than before.

= New pitch name array of just intonation =

					24 L	2 L	33 S	11 S				
				10 S	41 L	19 L	50 S	28 S	6 L	37 L	15 S	46 S
			49 S	27 S	5 L	36 L	14 S	45 S	23 L	1 L	32 S	
		35 L	13 S	44 S	22 L	0 L	31 S	9 S	40 L	18 L		
	21 L	52 L	30 S	8 S	39 L	17 L	48 S	26 S	4 L			
7 S	38 L	16 L	47 S	25 S	3 L	34 L	12 S	43 S				
				42 S	20 L	51 L	29 S					

Though in the next section this figure is explained, some basic items are described, prior to that.

☆☆ How to express intervals ☆☆

The pitch of tone is decided by its vibration frequency. When the pitches of 2 tones are compared, cent value is applied. This cent value is assumed to be Y and the ratio of 2 frequencies is assumed to be X. Then we get the next relation of numerical formula.

Y（cent）＝１２００（cent）×ｌｏｇX／ｌｏｇ２

The next explanation is just intonation, and then interval is always expressed by cent value.

For reference standardized interval cent value is presented next. La tone is common 440 Hz for both. Though cent value presented here is concrete numerical value, please note that it is often to show only interval quantity, whose unit is cent, and the concrete cent value is not presented. For example △ｐ (cent) of interval quantity is only presented.

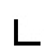
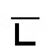
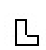
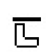
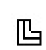
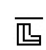
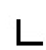
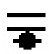
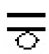
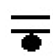
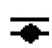
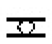
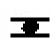
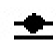
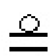
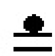
= Pythagorean intonation =

scale	do	re	mi	fa	so	la	ti	do
frequency (Hz)	260.74	293.33	330	347.65	391.11	440	495	521.48
interval (cent)	0	203.91	407.82	498.04	701.96	905.87	1109.8	1200

= Just intonation =

scale	do	re	mi	fa	so	la	ti	do
frequency (Hz)	264	297	330	352	396	440	495	528
interval (cent)	0	203.91	386.31	498.04	701.96	884.36	1088.27	1200

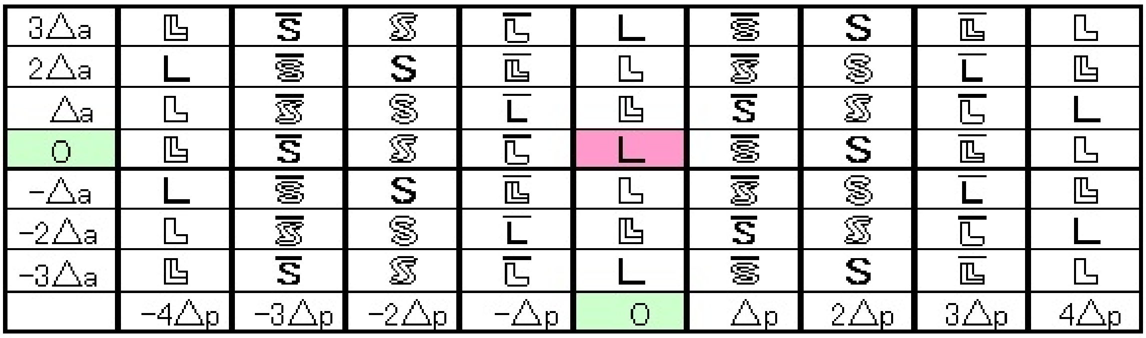
From this section music theory is described with a slightly different and new form, whose content is the same deservedly. Before this explanation the correspondence between code and pitch name is shown again, as you may not be accustomed to it. Though it is presented many times before, 12 tones and codes are often used after this. As they are basic items, please come familiar as much as possible. The note expression already presented is shown at the same time.

pitch name	1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB	1LW
code													
note													
syllable name	do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do

Among these codes,  fa,  so and  la are especially important.

There are Pythagorean intonation and just intonation. In music theory the regularity of tone arrangement is reviewed systematically for these intonations. This is the same as before. In the new exercise the next tone array is reviewed.

＝ The basic tone array in Pythagorean and just intonation ＝



Each column consists of 4 tone groups of line white tone, space black tone, space white tone and line black tone in the figure above. By rewriting a part of the figure we can get the following chart. If this chart is observed vertically, each column has a cycle of 1, 2 and 3 regularly.

	LW row	SB row	SW row	LB row	LW row	SB row	SW row	LB row	LW row
3△a	3	1	3	2	1	2	1	3	2
2△a	1	2	1	3	2	3	2	1	3
△a	2	3	2	1	3	1	3	2	1
0	3	1	3	2	1	2	1	3	2
−△a	1	2	1	3	2	3	2	1	3
−2△a	2	3	2	1	3	1	3	2	1
−3△a	3	1	3	2	1	2	1	3	2
	−4△p	−3△p	−2△p	−△p	0	△p	2△p	3△p	4△p

The tone array is very simple. Though the array is originally simple, we cannot see this simplicity, if we use conventional ＃& ♭. This point is not also convenient.

☆☆ What is △p ? ☆☆

Now I explain here what △p is. Interval between do tone and so tone is 7 half tones. As is well known, there is a difference of interval between those tones in equal temperament and Pythagorean intonation. Though interval of equal temperament is 700 (cents), that of Pythagorean temperament is adjusted to 700+△p (cents). This means that △p is the interval difference of 7 half tones in equal temperament and Pythagorean intonation. This difference is tiny. In Pythagorean intonation the ratio between the vibration frequency of so tone and that of do tone is 3 to 2. The vibration frequency of so tone is 1.5 times of that of do tone. The interval of both tone is 701.955 cents. Then △p becomes 1.955 cents.

As shown above the interval becomes △p (cents) higher than that of equal temperament in Pythagorean intonation. Therefore re tone which is further 700 cents higher than so tone is 1400 cents in equal temperament and 1400+2△p cents in Pythagorean intonation. By octave compensation we get 200+2△p cents for re tone. Pythagorean intonation is only the chain of tones which is made merely by the cycle operation of the increase of 700+△p. It is the same in minus side which is the decrease of 700+△p.
(By the way it is well known that the tone becomes “his” tone, if this cycle operation is executed 12 times. This tone is 23.46 cents higher than the original do tone. It is not rational that this tone becomes his tone. “his” tone is the relative of “h” tone and not the relative of “c” tone. It is rational, if this tone is the relative of “c” tone, as this tone is only 23.46 cents higher than the original do tone. This tone should be the relative of “c” tone to the last. It is not good that this tone becomes the relative of “h” tone, and not the relative of “c” tone. It is caused by using ♯& ♭. Use of ♯& ♭is a big problem, as it is made complicated.)

☆☆ What is △a ? ☆☆

Pythagorean intonation is the tone chain of the horizontal line, in which △p is related. In just intonation one more factor is added to it. This factor is △a. Then just intonation becomes an array of tones, which expands both to horizontal direction and vertical direction, in which △p and △a is related.

Though vertical columns are newly added in this intonation, it is similarly treated as an array of tones, as intervals of equal temperament and just intonation are also different. Interval between mi tone and do tone is 4 half tones. Then in equal temperament interval is 400 cents and the difference between equal temperament and just intonation is △a. In just intonation the ratio between the vibration frequency of mi tone and that of do tone is 5 to 4. The vibration frequency of mi tone is 1.25 times of that of do tone. The interval of both tones is 386.314 (cents). This interval is adjusted to 400-△a (cents). Then △a becomes 13.686 (cents).

Though lateral and vertical directions are thus decided, lateral direction is +△p and vertical direction is -△a. Both directions are different in sign, as +△p is plus and -△a is minus. The minus sign of vertical direction generates confusion at every time, as high level and low level of interval expression is always reverse. So it is devised not to generate confusion.

In order to solve this problem, 400-△a step of vertical direction is converted to 800+△a. Therefore the lateral direction is a step of 700+△p and vertical direction is a step of 800+△a. Then the integer ratio of the lateral direction is 3 to 2, and the integer ratio of the vertical direction is not 5 to 4, but becomes 8 to 5.

☆☆ Why can the vertical axis be assumed to be (800+△a) ? ☆☆

As described before interval between do tone and mi tone is 400-△a. Then it is explained just to make sure why the step of vertical direction is made 800+△a.

This is considered to be made the interval between do tone and mi tone which is an octave lower and the interval is -800-△a. Because the interval between do tone, which is an octave lower and -1200 cents, and mi tone, which is also an octave lower and -800-△a, becomes 400-△a. This means that we can make vertical direction step -800-△a toward minus direction. This means also that we can make vertical direction step 800+△a toward plus direction. This is simply an octave adjustment operation.

(note: The result is that lateral direction is 3/2=1.5 and 700 cents step and vertical direction is 8/5=1.6 and 800 cents step. Does it make us easy to remember them ?)

☆☆ Why △p and △a ? ☆☆

Two types of special quantity, that are △p and △a, are necessary. They are very important quantities for just intonation. It is not an exaggeration to say that Pythagorean intonation and just intonation are decided only by these 2 quantities. The content of △p and △a are already described, and I also explain why these quantities are expressed by using character p and character a. △p is a quantity which is applied in Pythagorean intonation and the initial letter of Pythagoras. △a is a quantity which is applied in just intonation. This quantity is due to the ratio of vibration frequencies of do and mi, which is 5 to 4. Archytas, who was in the first half of 4th century B.C., is said to propose the ratio of 5 to 4 first. Aristoxenos, who was in the latter half of 4th century B.C., is also said to study it in detail. Though Aristoxenos's initial letter is also a, Archytas is the pioneer and so "a" is his initial letter.

If necessary, △p is called Pythagorean quantity and △a is called Archytas's quantity. P=12△p and S=4△p+△a. These quantities are summarized in the next figure. S is a new quantity, which is described later.

	cent	case in which the quantity is used	quantity	formula
△p	1.955	horizontal direction・Pythagorean and just intonation	Pythagorean	
△a	13.686	vertical direction・just intonation	Archytas	
P	23.46	horizontal direction・Pythagorean and just intonation	Pythagorean comma	=12△p
S	21.506	vertical direction・just intonation	syntonic comma	=4△p+△a

(These explanations are made first by modifying the conventional concept. Especially △p and △a are a new description made by the author.)

Though tone array is totally explained roughly, the details of the reason why such tone array comes out need be described. As this is described separately in page 5 of volume 3 as details of just intonation tone arrangement, please refer to it.

☆☆ Explanation using integer and fraction ☆☆

Though tone array is explained by using interval cent value up to now, it can also be explained by using ratio, which is the fraction of integers, of vibration frequencies to a standard tone. But the integer becomes very large, which we never see usually before, and it becomes very troublesome in this way. The explanation of this method is omitted in this document, though it is possible to explain tone array with quite the same way by using only ratio of integers.

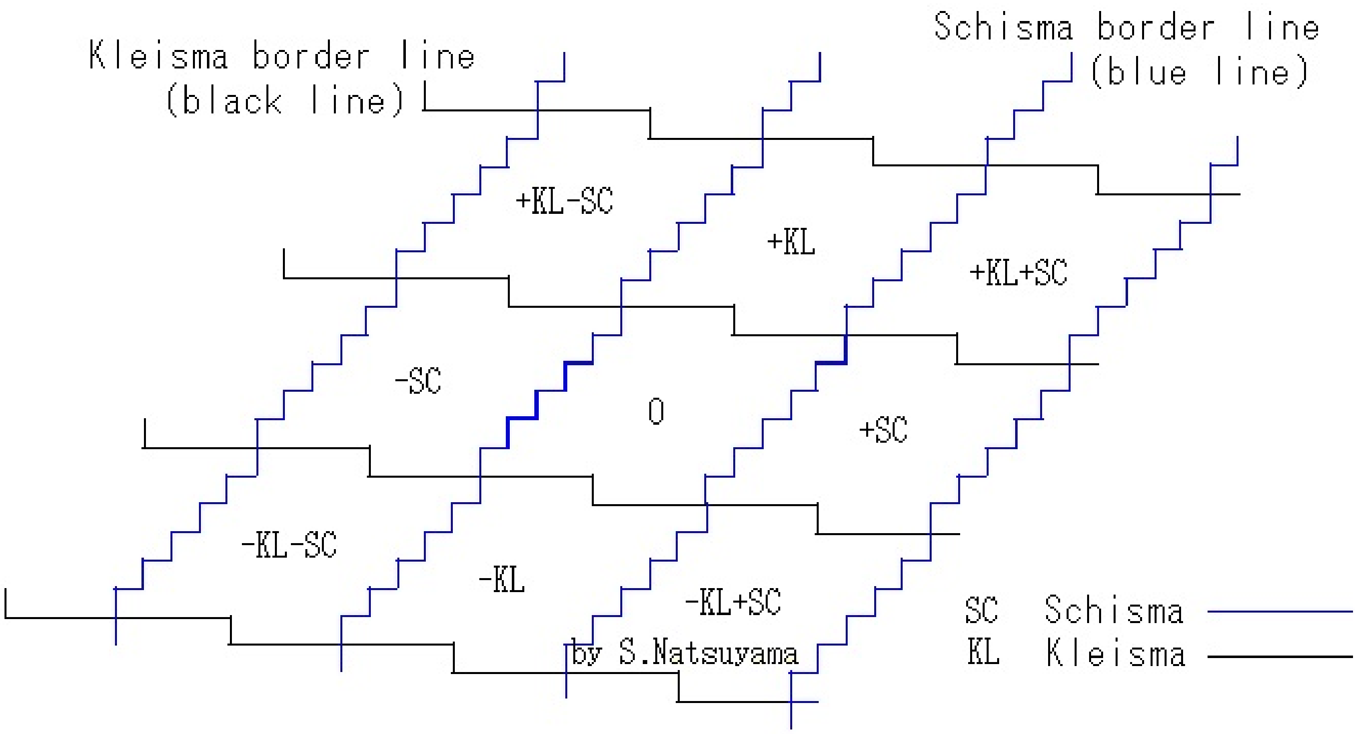
☆☆ Schisma and Kleisma ☆☆

As explained till now, innumerable numbers of just intonation tone exist as an array in an octave. There are infinite numbers of tone of just intonation naturally as physical phenomenon. It is a problem for practical use. Therefore just intonation tones are simplified by omitting tones which are similar with each other. Schisma and Kleisma are the means of classifying tones that can be omitted.

☆☆ Explanation of Schisma and Kleisma ☆☆

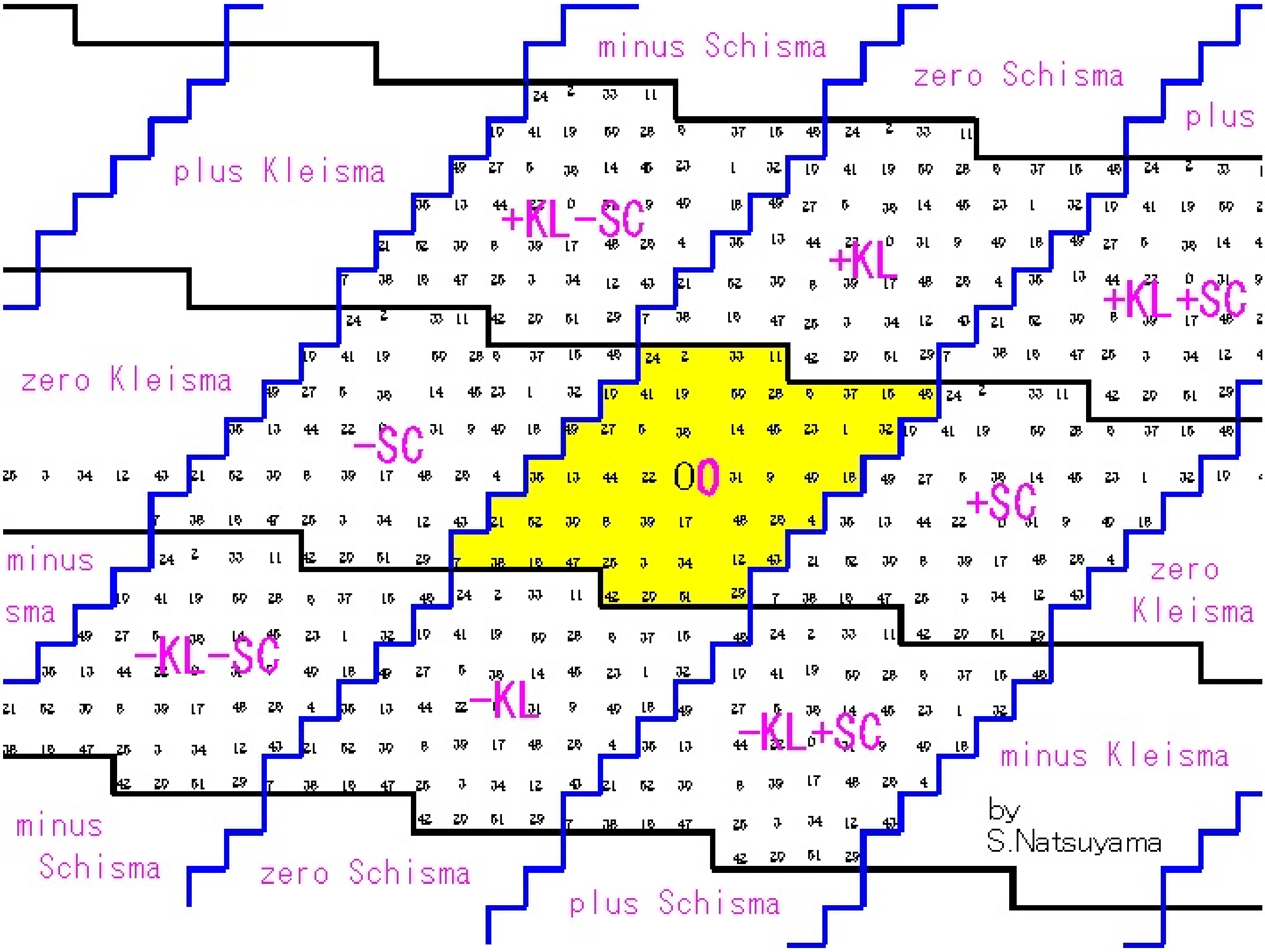
In the figure of next page SC means Schisma and KL means Kleisma. Each line shows the border line which classifies many scattered tones. With music theory Schisma border line neglects 1.95 cents error, and Kleisma border line neglects 8.11 cents error. Upper area than Kleisma border line, which is located in the above of origin, is plus Kleisma area and lower area than Kleisma border line , which is located in the beneath of origin, is minus Kleisma area. The tones in these two areas are omitted. Only the tones in the area in which origin is located are not omitted, and all other tones are omitted, because they are approximated tones. Similarly two Schisma border lines, which are located in the left side or the right side of the area where origin is located, classify the tones omitted. Similarly for Schisma border lines only the tones in the area in which origin is located are not omitted, and all other tones are omitted, because they are approximated tones.

= Border lines of Schisma and Kleisma =



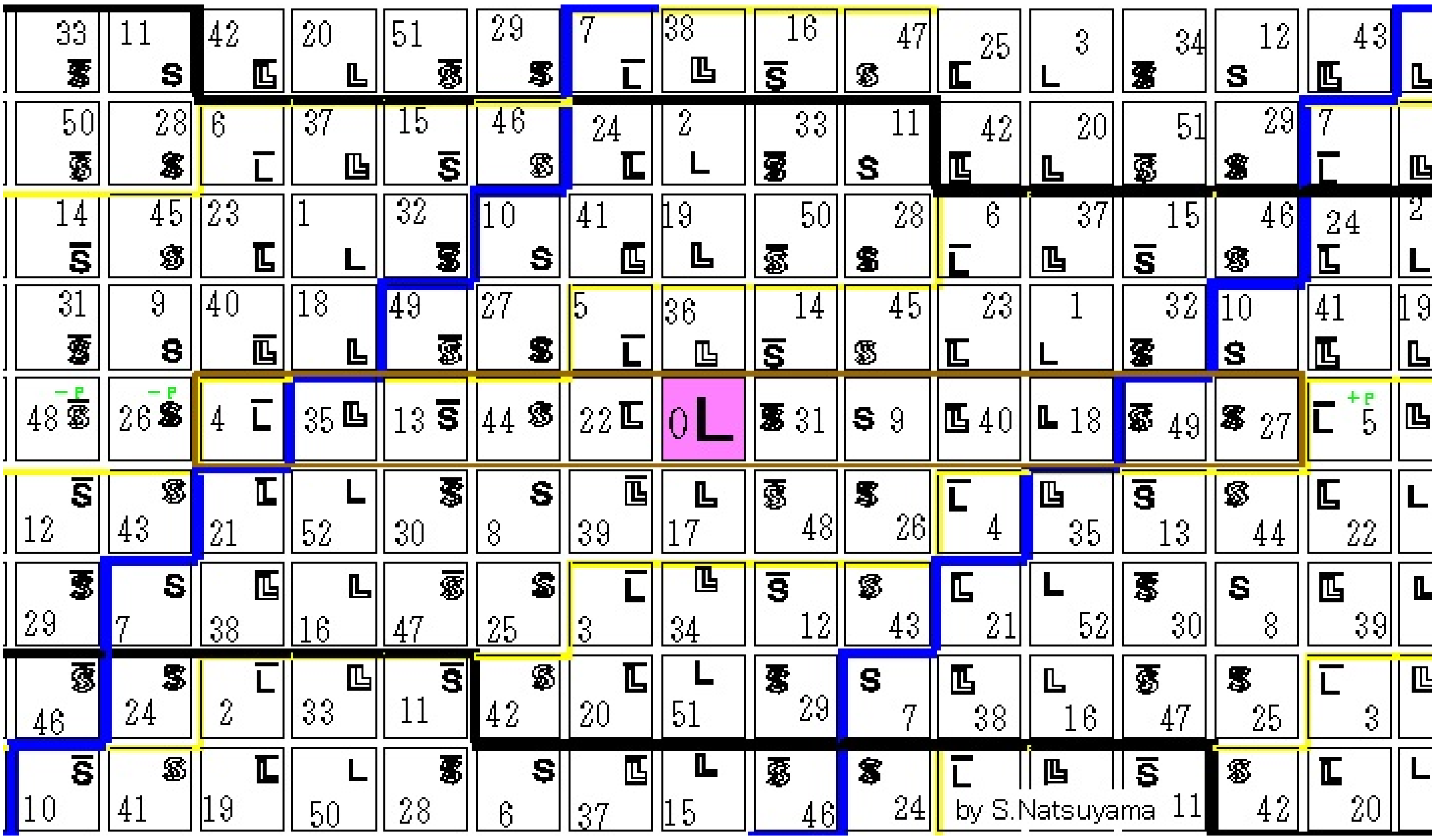
Schisma and Kleisma are explained in detail in page 9 of volume3 as details of Schisma and Kleisma. Please refer to it.

The figure of next page is to show that the area of scattered tones of just intonation is limited by Schisma and Kleisma. The tones, which locate only in zero Schisma and zero Kleisma of the yellow area, are those of just intonation. As mentioned above all other tones are omitted, because they are approximated tones.



In the next page is shown the magnified figure of just intonation tone array within zero Schisma area and zero Kleisma area.

＝ Figure of 53 tones of just intonation ＝



There are 53 tones of just intonation in the area belonging to zero Schisma area and belonging to zero Kleisma area, which can be called zero area. 12 tones in the area surrounded by a brown rectangle, which are in the straight line of lateral direction, are those of Pythagorean intonation. These tones become Pythagorean intonation fundamental 12 tones. All the other tones of these lines, which are located in the left side and the right side of the rectangle, are Pythagorean intonation tones.

☆☆ Just intonation 53 tone array ☆☆

The tone array of only 53 tones is shown in the figure of next page.

3△a						24 L	2 L	33 S	11 S				
2△a					10 S	41 L	19 L	50 S	28 S	6 L	37 L	15 S	46 S
△a				49 S	27 S	5 L	36 L	14 S	45 S	23 L	1 L	32 S	
0			35 L	13 S	44 S	22 L	0 L	31 S	9 S	40 L	18 L		
−△a		21 L	52 L	30 S	8 S	39 L	17 L	48 S	26 S	4 L			
−2△a	7 S	38 L	16 L	47 S	25 S	3 L	34 L	12 S	43 S				
−3△a					42 S	20 L	51 L	29 S					
	−6△p	−5△p	−4△p	−3△p	−2△p	−△p	0	△p	2△p	3△p	4△p	5△p	6△p

☆☆ Rearrangement of the array ☆☆

Though this array is clearer than before, there are still some problems. In order to make it clearer, this diagram is arranged as shown in the next page. In this diagram the yellow lines of shapes of stairs are made straight lines, which appear in the figure before but one. This modification makes horizontal axis changed by the cycle of 12△p and vertical axis changed by the cycle of 4△p+△a. As described before P is Pythagorean comma and equals to 12△p, and S is syntonic comma and equals to 4△p+△a. These two are very important quantities which become unit intervals in just intonation. (Though this rearrangement of the array seems simple, it is coordinate transformation in mathematics from orthogonal coordinates to oblique coordinates.)

When you want to know details of 53 array, you can see them as details of figure of just intonation in page 11 of volume 3. Please refer to it.

＝ Expression by using Pythagorean comma and syntonic comma ＝

syntonic comma		2LB	1LW	2SB	1SW	3LB	2LW	3SB	2SW	1LB	3LW	1SB	3SW	2LB	1LW	2SB	1SW	3LB	2LW	3SB	2SW	1LB	3LW	1SB	3SW	2LB	1LW	2SB
plus	+3S	24	2	33	11																							
	+2S				10	41	19	50	28	6	37	15	46															
	+S							49	27	5	36	14	45	23	1	32												
zero	0										35	13	44	22	**0**	31	9	40	18									
minus	−S													21	52	30	8	39	17	48	26	4						
	−2S																7	38	16	47	25	3	34	12	43			
	−3S																								42	20	51	29
		−P minus Pythagorean comma								0 zero Pythagorean comma												+P plus Pythagorean comma						

The figure above is separated to 3 parts of minus Pythagorean comma region in the left, no Pythagorean comma region in the center and plus Pythagorean comma region in the right. In the next figure these 3 regions are overlapped and the columns, in which there are the same kinds of tones, are collected to one column. In the figure the left side part of the figure above comes in upper part of this figure. The central part comes in central part and the right side part comes in lower part. By this collection all the same tones (for example 1LW) come in the same column. And 12 columns are made arranged. The numbers in the figure are the numbers of 53 just intonation tones.

This figure shows that each tone is expressed by using both Pythagorean comma and syntonic comma, and that for example 1LW tones are 5 tones, and they are expressed as **L**P-3S, **L**-S, **L**, **L** S and **L**-P+3S.

＝ The figure being collected to 12 tone columns ＝

syntonic comma		1LB	3LW	1SB	3SW	2LB	1LW	2SB	1SW	3LB	2LW	3SB	2SW
plus	+3S					24 −P+3S	2 −P+3S	33 −P+3S	11 −P+3S				
	+2S	6 2S	37 2S	15 2S	46 2S				10 −P+2S	41 −P+2S	19 −P+2S	50 −P+2S	28 −P+2S
	+S	5 S	36 S	14 S	45 S	23 S	1 S	32 S				49 −P+S	27 −P+S
0	0		35 0	13 0	44 0	22 0	**0** 0	31 0	9 0	40 0	18 0		
minus	−S	4 P−S				21 −S	52 −S	30 −S	8 −S	39 −S	17 −S	48 −S	26 −S
	−2S	3 P−2S	34 P−2S	12 P−2S	43 P−2S				7 −2S	38 −2S	16 −2S	47 −2S	25 −2S
	−3S				42 P−3S	20 P−3S	51 P−3S	29 P−3S					

The next figure is obtained, when each tone is rearranged in interval order, after octaves are compensated. By this rearrangement 53 tones of just intonation are obtained.

＝ The figure of tones arranged in interval order ＝

syntnic comma		1LW L	1LB	1SW S	1SB	2LW	2LB	2SW	2SB	3LW	3LW	3SW	3SB	1LW L
plus	+3S	2 -P+3S		11 -P+3S			24 -P+3S		33 -P+3S					
	+2S		6 2S	10 -P+2S	15 2S	19 -P+2S		28 -P+2S		37 2S	41 -P+2S	46 2S	50 -P+2S	
	+S	1 S	5 S		14 S		23 S	27 -P+S	32 S	36 S		45 S	49 -P+S	
0	0	0 0		9 0	13 0	18 0	22 0		31 0	35 0	40 0	44 0		53 0
minus	-S		4 P-S	8 -S		17 -S	21 -S	26 -S	30 -S		39 -S		48 -S	52 -S
	-2S		3 P-2S	7 -2S	12 P-2S	16 -2S		25 -2S		34 P-2S	38 -2S	43 P-2S	47 -2S	
	-3S						20 P-3S		29 P-3S			42 P-3S		51 P-3S
L(C) major		do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do

The figure above is point symmetry and S tone is the center. This means that S tone increases in the order of P-3S, -S, 0, S, ・・・・, and L tone decreases in the order of -P+3S, S, 0, -S, ・・・・. They are the same rate of increasing and decreasing, though the sign is reverse. Both in the upper direction and the lower direction the tones change at the same rate up to L tone of No.53 and also down to L tone of No.0. This shows point symmetry.

Chapter 3 Pythagorean intonation and just intonation

☆☆ Positioning of equal temperament and just intonation ☆☆

As for positioning of equal temperament and just intonation, just intonation was originally used, and equal temperament began to be gradually used. Equal temperament is mainly used now. Just intonation did not yet go out of use, as it is main use in the field of stringed instruments.

The relation between equal temperament and just intonation is considered that there are equal temperament and just intonation, and that they coexist merely independently. The author considers that it is better to clarify the relation between equal temperament and just intonation. It should be the relation as described below.

First of all the fundamental temperament should be equal temperament, and not just intonation.

Just intonation is the modification or the application of equal temperament. Equal temperament is the starting point. When equal temperament is taken into consideration at first, just intonation is well understood.

In the field of stringed instruments Pythagorean intonation becomes main naturally. Though it becomes so historically and inevitably, it is meaningful that the basis of Pythagorean intonation is equal temperament. Though it is different from music history, we should consider that just intonation can be obtained only by the modificatin of equal temperament. As stated till now, it is easy to consider that the starting point is not Pythagorean intonation but equal temperament even in the stringed instruments.

It becomes that the tones of equal temperament and Pythagorean intonation are well explained by stopping the use of ＃& ♭. It is not ＃& ♭ but Pythagorean comma and syntonic comma what is really necessary to Pythagorean intonation and just intonation. In page 98 of volume 3 you can see details about positioning of equal temperament and just intonation, please refer to it.

By the way there are some kinds of just intonation. If they are classified into 3, the classification becomes as shown in the next table.

intonation	frequency relation among intonation forming tones	limit
1. Pythagorean	multipied by the fraction which make mutiples of 2 or 3 to the numerator and the denominator	3－limit
2. just	multipied by the fraction which make mutiples of 2,3 or 5 to the numerator and the denominator	5－limit
3. others	multipied by the fraction which make mutiples of 2,3, 5,7 or more to the numerator and the denominator	7－limit or limit of more than 7

In this classification 7-limit or more is not so often used. Though there are very numerous integers theoretically, clearly harmonized chords are not expected, if the number of integer becomes big.

☆☆ Pythagorean intonation ☆☆

Pythagorean intonation is shown in the next page. In the table each tone of Pythagorean intonation is decided by depending on the increase and the decrease of △p.

△p	－5	－4	－3	－2	－1	0	＋1	＋2	＋3	＋4	＋5	＋6
syllable name	dor	soll	rem	lat	fa	do	so	re	la	mi	ti	fas

In the next table code of pitch name is presented. Though the next tables are separated to 3, it is one table, which is a bridging laterally aligned. As the table is too long, it is separated, owing to the increase of △p from minus to plus.

△p	−17	−16	−15	−14	−13	−12	−11	−10	−9	−8	−7	−6
音名	L̄-P	𝕃-P	S̄-P	𝕊-P	ᒪ̄-P	L-P	𝔖̄-P	S-P	𝕃̄-P	ᒪ-P	𝕊̄-P	𝔖-P

△p	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6
音名	L̄	𝕃	S̄	𝕊	ᒪ̄	L	𝔖̄	S	𝕃̄	ᒪ	𝕊̄	𝔖

△p	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
音名	L̄P	𝕃P	S̄P	𝕊P	ᒪ̄P	LP	𝔖̄P	SP	𝕃̄P	ᒪP	𝕊̄P	𝔖P

☆☆ Pythagorean intonation scale ☆☆

The scale of Pythagorean intonation is in the next table. This table is L major (C major). Code standard expression means that code is standard and half tones. For example L̄ means 100 cents and S means 200 cents. On the other hand code Pythagorean intonation expression means that code includes not only half tones but also quantity compensated with △p, which is the tone of Pythagorean intonation and p is attached just before code.

scale	do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do
note													
code standard expression	L	L̄ −5△p	S 2△p	S̄ −3△p	𝕃 4△p	ᒪ̄ −△p	𝕊 6△p	𝔖̄ △p	𝕃 −4△p	𝕃̄ 3△p	𝕊 −2△p	𝔖̄ 5△p	L
interval shift of neighboring tone from 100 cents		−5△p	7△p	−5△p	7△p	−5△p	7△p	−5△p	−5△p	7△p	−5△p	7△p	−5△p
code Pythagorean intonation expression	pL	pL̄	pS	pS̄	p𝕃	pᒪ̄	p𝕊	p𝔖̄	p𝕃	p𝕃̄	p𝕊	p𝔖̄	pL
interval (cent)	0	90.2	203.9	294.1	407.8	498.0	611.7	702.0	792.2	905.9	996.1	1109.8	1200

Interval shift of neighboring tone from 100 cents or 200 cents becomes regularly the periodic cycle of -5△p and 7△a. But only the interval shifts among fa tone, fas tone and so tone are to be noted that they are irregular. Moreover it is noted that 𝕊+6△p tone (fas) is the center and both tones, that are becoming higher and that are becoming lower, are changing with the same value of △p, though their plus and minus signs are quite reverse. This shows that the tone chain is symmetry in which 𝕊 +6△p tone is the center.

L (C) major is in the next table, if only **L** major scale is taken out of the chain.

scale	do	re	mi	fa	so	la	ti	do
note								
code standard expression	**L**	**S**+2△p	𝕃+4△p	L̄−△p	S̄+△p	𝕃̄+3△p	S̿+5△p	**L**
interval shift of neighboring tone from 100 cents		2△p	2△p	−5△p	2△p	2△p	2△p	−5△p
code Pythagorean intonation expression	p**L**	p**S**	p𝕃	pL̄	pS̄	p𝕃̄	pS̿	p**L**

In this case interval shift of neighboring tone from 100 cents or 200 cents becomes 2△p (3.91 cents) and -5△p (-9.78 cents). In tuning of stringed instruments of equal temperament whole tones are to be shifted down to about 4 cents and half tones are to be shifted up to about 10 cents, as compared with Pythagorean intonation tuning. This rule is clearly shown in the table.

In other keys interval shift of each tone is the same. **S** (D) major and 𝕃̄ (A) major are shown below, which are often used in stringed instruments. (These keys can be also obtained by using the same 12 tones used in **L** key.)

S major scale	do	re	mi	fa	so	la	ti	do
note								
code standard expression	**S**+2△p	𝕃+4△p	𝕊+6△p	S̄+△p	𝕃̄+3△p	S̿+5△p	L̄+7△p	**S**+2△p
interval shift of neighboring tone from 100 cents		2△p	2△p	−5△p	2△p	2△p	2△p	−5△p
code Pythagorean intonation expression	p**S**	p𝕃	p𝕊P	pS̄	p𝕃̄	pS̿	pL̄P	p**S**

L major scale	do	re	mi	fa	so	la	ti	do
note								
code standard expression	L+3Δp	S+5Δp	L+7Δp	S+2Δp	L+4Δp	S+6Δp	L+8Δp	L+3Δp
interval shift of neighboring tone from 100 cents		2Δp	2Δp	−5Δp	2Δp	2Δp	2Δp	−5Δp
code Pythagorean intonation expression	pL	pS	pLP	pS	pL	pS	pLP	pL

The details of other keys of Pythagorean intonation are described in page 14 of volume 3 as Pythagorean intonation. Please refer to it.

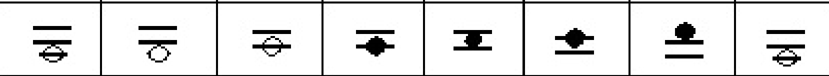
☆☆ Just intonation ☆☆

12 tones of just intonation are expressed as the compensation with Δp and Δa as well as the case of Pythagorean intonation and they are shown in the next table. In this case code just intonation expression is used as well as Pythagorean intonation. This code means not only half tone but also just intonation tone which is expressed with Δp and Δa. But every "j" is omitted in the description after this, as in many cases it is clear contextually. (As it is convenient that the tone of just intonation is expressed with using the tone of Pythagorean intonation, there are practically very few cases of code just intonation expression.)

L (C) major	do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do
note													
code just intonation expression	jL	jL	jS	jS	jL	jL	jS	jS	jL	jL	jS	jS	jL
number	0	5	9	14	17	22	26	31	36	39	44	48	53
code standard expression	L	L −Δp +Δa	S +2Δp	S +Δp +Δa	L −Δa	L −Δp	S +2Δp −Δa	S +Δp	L +Δa	L −Δp −Δa	S −2Δp	S +Δp −Δa	L
interval (cent)	0	111.7	203.9	315.6	386.3	498.0	590.2	702.0	813.7	884.4	996.1	1088.3	1200

The reason, why 12 tones of just intonation is selected like this, is described as “Explanation on the selection of just intonation 12 tones” in page 17 of volume 3. Please refer to it if you are interested.

Only the scale of just intonation is as follows.

L (C) major	do	re	mi	fa	so	la	ti	do
note								
code just intonation expression	jL	jS	jL	jL	jS	jL	jS	jL
number	0	9	17	22	31	39	48	53
code standard expression	L	S +2△p	L −△a	L −△p	S +△p	L −△p −△a	S +△p −△a	L

☆☆ Simplification of just intonation tone expression (new pitch name) ☆☆

Up to now 53 tones of just intonation are described in detail and there is a problem in the expression method of just intonation. As the interval of just intonation is rather complicated, the expression is also complicated. As described before, however, the tones of just intonation are limited, we can identify sufficiently their tones with ease.

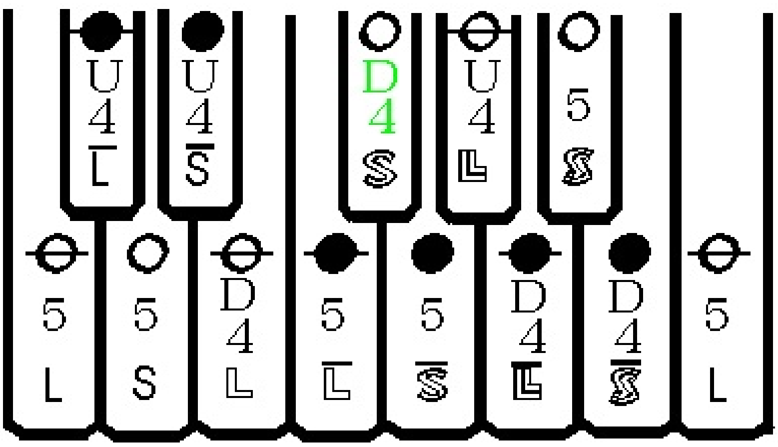
Therefore we can identify them by simplifying their pitch names. The simplified pitch names are shown in the next figure. Instead of using Pythagorean comma or syntonic comma, A and B are used. In just intonation interval of the neighboring tones are basically 1 syntonic comma. Then A is applied, if interval is 1 syntonic comma, and B is applied, if interval is 2 syntonic commas. Practically there is a slight difference from 1 syntonic comma in interval of neighboring tones, owing to Schisma and Kleisma conversion, as already stated. This slight difference is neglected as quasi-syntonic comma and quasi-syntonic comma is regarded as 1 unit of a syntonic comma.

Quasi-syntonic comma is sometimes equal to syntonic comma and sometimes a little different. Interval of 1 quasi-syntonic comma is expressed as A and that of 2 quasi-syntonic commas is expressed as B. If A is maximum and there is no B, A is expressed as U. If -A is minimum and there is no -B similarly, -A is expressed as -D. (U and D come from utmost and downmost respectively.) It is the discrimination of half tones, which have 5 tones and which have 4 tones within a half tone. Therefore U is the same as A and -D is same as -A. In other words both U and D mean A. Details of quasi-syntonic comma are described as detailed explanation on quasi-syntonic comma in page 20 of volume 3. Please refer to it.

pseudo-syntonic comma			1LW	1LB	1SW	1SB	2LW	2LB	2SW	2SB	3LW	3LB	3SW	3SB	1LW
			L	L̄	S	S̄	𝕃	𝕃̄	𝕊	𝕊̄	𝕃	𝕃̄	𝕊	𝕊̄	L
plus	+2	+B	2 B		11 B		19 B	24 B	28 B	33 B		41 B	46 B	50 B	
	+1	+A	1 A	6 U	10 A	15 U	18 A	23 A	27 A	32 A	37 U	40 A	45 A	49 A	
0	0	0	0 0	5 0	9 0	14 0	17 0	22 0	26 0	31 0	36 0	39 0	44 0	48 0	53 0
minus	−1	−A		4 −A	8 −A	13 −A	16 −D	21 −A	25 −D	30 −A	35 −A	38 −D	43 −A	47 −D	52 −A
	−2	−B		3 −B	7 −B	12 −B		20 −B		29 −B	34 −B		42 −B		51 −B
L (C) major			do	dor	re	rem	mi	fa	fas	so	soll	la	lat	ti	do
total notes U·D			5	4 U	5	4 U	4 D	5	4 D	5	4 U	4 D	5	4 D	5

In the figure by making the column of 𝕊 tone the center, each tone is point symmetry. 𝕊 tone is increasing by -B, -A, 0, ・・・・・・ and 𝕃̄ tone is decreasing by B, A, 0, ・・・・・・, as the signs are reverse and the values are the same.

The number of tones shown in this figure can be shown in the next figure in another style. The tone numbers in just intonation is in the figure. 𝕊 tones expressed in green color is the center of all tones. The tones go up in the order of 5, U4, D4, ・・・・・ and the tones go down in the order of 5, D4, U4, ・・・・・. Though the plus minus is reverse, numbers of tones are the same.



Just intonation tones can be explained in these new methods. Therefore just intonation 53 tones are summarized as shown in the next table.

pitch name	1LW L	1LB L̄	1SW S	1SB S̄	2LW 𝕃	2LB 𝕃̄	2SW 𝕊	2SB 𝕊̄	3LW 𝓛	3LB 𝓛̄	3SW 𝓢	3SB 𝓢̄	1LW L
+2	+B		+B		+B	+B	+B	+B		+B	+B	+B	+B
+1	+A	+U	+A	+U	+A	+A	+A	+A	+U	+A	+A	+A	+A
0	0	0	0	0	0	0	0	0	0	0	0	0	0
−1	−A	−A	−A	−A	−D	−A	−D	−A	−A	−D	−A	−D	−A
−2	−B	−B	−B	−B		−B		−B	−B		−B		−B

In the next table just intonation tones arranged in numerical sequence are shown. In this table pitch names already described are also shown.

＝ Table of 53 tones of just intonation ＝

No.	pitch name	name call	No.	pitch name	name call	No.	pitch name	name call	No.	pitch name	name call	No.	pitch name	name call
51	LP-3S	L−B	9	S	S	20	𝕃̄P-3S	𝕃̄−B	31	𝕊̄	𝕊̄	42	𝓢P-3S	𝓢−B
52	L-s	L−A	10	S-P+2S	SA	21	𝕃̄-s	𝕃̄−A	32	𝕊̄s	𝕊̄A	43	𝓢P-2S	𝓢−A
0(53)	L	L	11	S-P+3S	SB	22	𝕃̄	𝕃̄	33	𝕊̄-P+3S	𝕊̄B	44	𝓢	𝓢
1	Ls	LA	12	S̄P-2S	S̄−B	23	𝕃̄s	𝕃̄A	34	𝓛P-2S	𝓛−B	45	𝓢s	𝓢A
2	L-P+3S	LB	13	S̄	S̄−A	24	𝕃̄-P+3S	𝕃̄B	35	𝓛	𝓛−A	46	𝓢2s	𝓢B
3	L̄P-2S	L̄−B	14	S̄s	S̄	25	𝕊-2s	𝕊−D	36	𝓛s	𝓛	47	𝓢̄-2s	𝓢̄−D
4	L̄P-S	L̄−A	15	S̄2s	S̄U	26	𝕊-s	𝕊	37	𝓛2s	𝓛U	48	𝓢̄-s	𝓢̄
5	L̄s	L̄	16	𝕃-2s	𝕃−D	27	𝕊-P+S	𝕊A	38	𝓛̄-2s	𝓛̄−D	49	𝓢̄-P+S	𝓢̄A
6	L̄2s	L̄U	17	𝕃-s	𝕃	28	𝕊-P+2S	𝕊B	39	𝓛̄-s	𝓛̄	50	𝓢̄-P+2S	𝓢̄B
7	S-2s	S−B	18	𝕃	𝕃A	29	𝕊̄P-3S	𝕊̄−B	40	𝓛̄	𝓛̄A			
8	S-s	S−A	19	𝕃-P+2S	𝕃B	30	𝕊̄-s	𝕊̄−A	41	𝓛̄-P+2S	𝓛̄B			

The array of 53 tones is shown in the next figure. The array is point symmetry whose center is L tone. (𝕊 tone is excluded.) The meaning of point symmetry is further described in page 21 of volume 3. Please refer to it.

					24 𝕃̄B	2 LB	33 𝕊̄B	11 SB				
				10 SA	41 𝓛̄B	19 𝕃B	50 𝓢̄B	28 𝕊B	6 L̄U	37 𝓛U	15 S̄U	46 𝓢B
			49 𝓢̄A	27 𝕊A	5 L̄	36 𝓛	14 S̄	45 𝓢A	23 𝕃̄A	1 LA	32 𝕊̄A	
		35 𝓛-A	13 S̄-A	44 𝓢	22 𝕃̄	0 L	31 𝕊̄	9 S	40 𝓛̄A	18 𝕃A		
	21 𝕃̄-A	52 L-A	30 𝕊̄-A	8 S-A	39 𝓛̄	17 𝕃	17 48𝓢̄	26 𝕊	4 L̄-A			
7 S-B	38 𝓛̄-D	16 𝕃-D	47 𝓢̄-D	25 𝕊-D	3 L̄-B	34 𝓛-B	12 S̄-B	43 𝓢-A				
				42 𝓢-B	20 𝕃̄-B	51 L-B	29 𝕊̄-B					

☆☆ A sequence of 7 half tones ☆☆

In the array of 53 tones of just intonation 7 half tone sequence is obtained. The sequence is made up by connecting each 7 half tone in interval order. This sequence starts from **L** tone and ends also to **L** tone, after connecting 53 tones. Next is a sequence of 7 half tones.

L~𝕊̄~S~𝕃̄A~𝕃A~Schisma~𝒮̄A~𝕊A~L̄~𝕃~S̄~𝒮A~𝕃̄A~LA~𝕊̄A~Schisma~SA~𝕃̄B~𝕃B~𝒮̄B~𝕊B~L̄U~𝕃U~S̄U~𝒮B~Schisma~𝕃̄B~LB~𝕊̄B~SB ~Kleisma~𝒮-B~𝕃̄-B~L-B~𝕊̄-B~Schisma~S-B~𝕃̄-D~𝕃-D~𝒮̄-D~𝕊-D~L̄-B~𝕃-B ~S̄-B~𝒮-A~Schisma~𝕃̄-A~L-A~𝕊̄-A~S-A~𝕃̄~𝕃~𝒮̄~𝕊~L̄-A~Schisma~𝕃-A~S̄-A~𝒮~𝕃̄~L

☆☆ 12 tones of equal temperament and fundamental 12 tones of Pythagorean intonation ☆☆

First of all there are 12 tones of equal temperament. Similarly there are 12 tones of Pythagorean intonation. 12 tones of Pythagorean intonation are the fundamental tones not only for Pythagorean intonation but also for just intonation. Therefore it is very convenient to specify these 12 tones as fundamental 12 tones of Pythagorean intonation.

All of these 12 tones are those of Pythagorean intonation and some of them are also those of just intonation. By using these fundamental 12 tones, all tones of Pythagorean intonation and just intonation can be expressed, with attaching Pythagorean commas and syntonic commas. This means that fundamental 12 tones of Pythagorean intonation are the basic tones for expressing both tones of Pythagorean intonation and tones of just intonation.

Every Pythagorean intonation key can be also obtained only by using fundamental 12 tones of Pythagorean intonation. Then wolf problem occurs.

equal temperament		L	L̄	S	S̄	𝕃	𝕃̄
interval (cent)		0	100	200	300	400	500
numbers of half tones		0	1	2	3	4	5
Pythagorean intonation fundamental 12 tones	code Pythagorean intonation expression	pL	pL̄	pS	pS̄	p𝕃	p𝕃̄
	code standard expression	L	L̄-5△p	S+2△p	S̄-3△p	𝕃+4△p	𝕃̄-△p
	interval (cent)	0	100-5△p	200+2△p	300-3△p	400+4△p	500-△p
	tones included in 53 tones of just intonation	included tone		included tone			included tone

equal temperament		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
interval (cent)		600	700	800	900	1000	1100
numbers of half tones		6	7	8	9	10	11
Pythagorean intonation fundamental 12 tones	code Pythagorean intonation expression	p[illegible]	p[illegible]	p[illegible]	p[illegible]	p[illegible]	p[illegible]
	code standard expression	[illegible]+6△p	[illegible]+△p	[illegible]−4△p	[illegible]+3△p	[illegible]−2△p	[illegible]+5△p
	interval (cent)	600+6△p	700+△p	800−4△p	900+3△p	1000−2△p	1100+5△p
	tones included in 53 tones of just intonation		included tone			included tone	

☆☆ How to count and express intervals ☆☆

As mentioned above Pythagorean comma and syntonic comma play a very important role in expressing interval of just intonation. Here interval expression in general is summarized. Interval is decided by the next 3 items.

① Number of half tone

② Number of Pythagorean comma

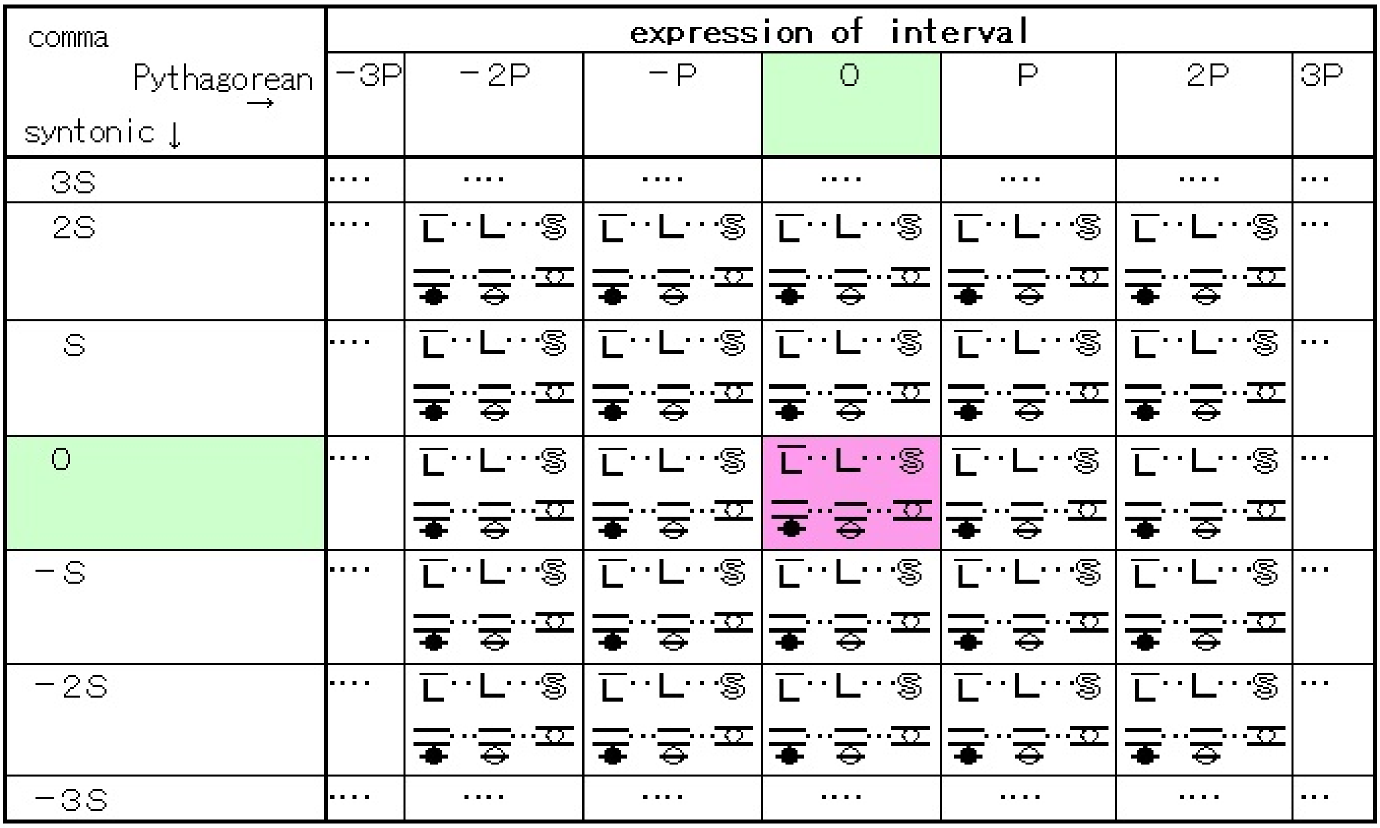
③ Number of syntonic comma

Interval expression is naturally different, if intonation differs. For example in equal temperament only half tone is used. Items, which decide interval expression, are summarized in the next table.

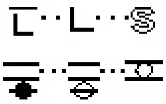
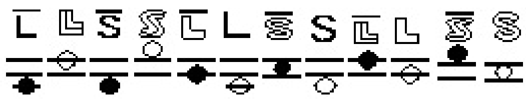
temperament	items which decide interval
equal temperament	number of half tones
Pythagorean intonation	number of half tones and Pythagorean comma
just intonation	number of half tones , Pythagorean comma and syntonic comma

In both Pythagorean intonation and just intonation, Pythagorean intonation fundamental 12 tones are used instead of half tones. In these tones it is only necessary to know how the tones are separated from Pythagorean intonation fundamental 12 tones. Then the interval expression becomes as follows. Half tones are expressed first. Moreover the numbers of Pythagorean comma are plotted in lateral direction and the numbers of syntonic comma are plotted in vertical direction. The tones are summarized in the next diagram. Practical interval value can be obtained by calculation. Please calculate interval (cents) with opening the page for calculation. Open the page. （http://km87290.web.fc2.com/htdocs/English/page20.html） Open this page similarly hereafter, when the calculation of interval is necessary.

Though the figure is very broad area, practical Pythagorean comma is -1, 0 or +1 and the mark is -P, no mark, +P. And syntonic comma is from 0 to about plus minus 3 in real use.



Omitted mark in the each block of the figure above

 shows the omitted shape of  .

These show codes and the note positions of Pythagorean intonation fundamental 12 tones.

pL etc. are precise expression, as they are Pythagorean intonation. But in order to prevent complexity, p is omitted. Therefore this figure becomes as shown in the next page.

comma Pythagorean → syntonic ↓	expression of interval						
	−3P	−2P	−P	0	P	2P	3P
3S	···	····	····	····	····	····	···
2S	···	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	···
S	···	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	···
0	···	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	···
−S	···	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	···
−2S	···	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	fundamental tones of P. intonation	···
−3S	···	····	····	····	····	····	···

(P. intonation means Pythagorean intonation.)

Interval expression of each tone is decided with the rule of this table. All tones can be expressed with this rule. Therefore interval of all tones can be expressed by using their half tone, Pythagorean comma and syntonic comma. In the next table are shown some interval examples of 2 tones. They are **L** and **S**.

half tones	Pythagorean comma	syntonic comma	interval
0	0	0	**L**
0	1	0	**L**P
0	−1	2	**L**−P+2S
7	0	0	**S**
7	−1	0	**S**−P
7	1	2	**S**P+2S

Though there is no problem in expressing interval of equal temperament and Pythagorean intonation, just intonation is rather complicated, as syntonic comma is contained. But as the numbers of just intonation tones are limited, they are easily discriminated, even if their names are simplified without using syntonic comma. Therefore their expressions are simplified in the special way, as described before. Pitch name call of just intonation is already presented in the table of 53 tones in page 32. Pitch name expresses also interval at the same time. Therefore pitch name call expresses interval, too.

☆☆ Improvement of pitch name expression ☆☆

Compared with conventional pitch names, new pitch name expression is improved. New pitch name expression is made more clear than that of conventional pitch names. The inconvenient points of conventional pitch name are explained as ambiguous and inconvenient pitch name in page 22 of volume 3. If you are interested in them, please refer to it.

(to be continued to volume 2)

Content (volume 2)

Content (Volume 3)

♯&♭ are not necessary at all to music (Volume 1)

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